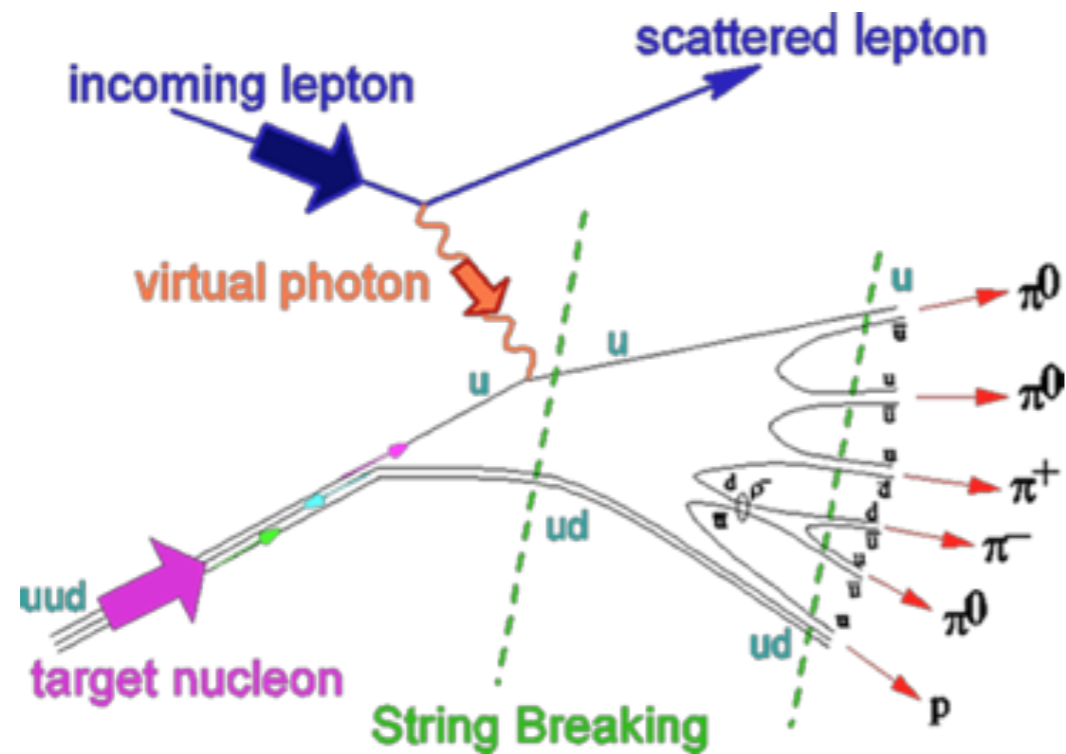


Matching TMD factorization and collinear factorization



Joint CTEQ Meeting and 7th International Conference
on Physics Opportunities at an EIC (POETIC 7)

Temple University

Leonard Gamberg



Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

Outline

- ◆ Review work on improved implementation for combining transverse-momentum-dependent (TMD) factorization and collinear factorization in semi-inclusive DIS
- ◆ The result is a modified version of the “ $W+Y$ ” prescription traditionally used in the Collins-Soper-Sterman (CSS) formalism [Collins Soper Sterman NPB 1985](#)
- ◆ Address the “standard matching prescription” traditionally used in the CSS formalism relating low and high q_T behavior of cross section @ moderate Q
- ◆ In particular the role of Y term matching of low and high q_T behavior of cross section @ moderate Q
- ◆ **Introduce method to combine TMD and Collinear Factorization formalism**
- ◆ We briefly discuss how an EIC could help to further our study of matching between the TMD approach and collinear factorization.

Comments

- ◆ The standard $W + Y$ prescription was arranged to apply also for intermediate q_T ; in particular it keeps full accuracy when $m \ll q_T \ll Q$, a situation in which both pure TMD and pure collinear factorization have degraded accuracy
- ◆ It also did not specifically address the issue of matching to collinear factorization for the cross section integrated over q_T
- ◆ With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of $1/Q$.
- ◆ Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used.

Start w/ review of CSS $W + Y$ construction.

- The CSS formalism separates the cross section into a sum of two terms W & Y such that $W+Y$ give the cross section up to an error that relative to the cross section is power suppressed as $(m/Q)^c$ where $c > 0$

$$\frac{d\sigma(q_T, Q)}{d^2q_T dQ \dots} \equiv \Gamma(q_T, Q) - \text{shorthand notation for cross section}$$

$$\Gamma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

- W describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$
- W is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence
- The Y -term is described in terms of “collinear approximation” to the cross section: it is the correction term for large $q_T \sim Q$

The $W + Y$ construction

$$\Gamma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

- The CSS construction of $W + Y$ and the specific approximations applied, thru the operations T_{TMD} and T_{coll} work only in the regions $q_T \ll Q$ and $q_T \sim Q$ respectively, which we emphasize by the range of the argument above

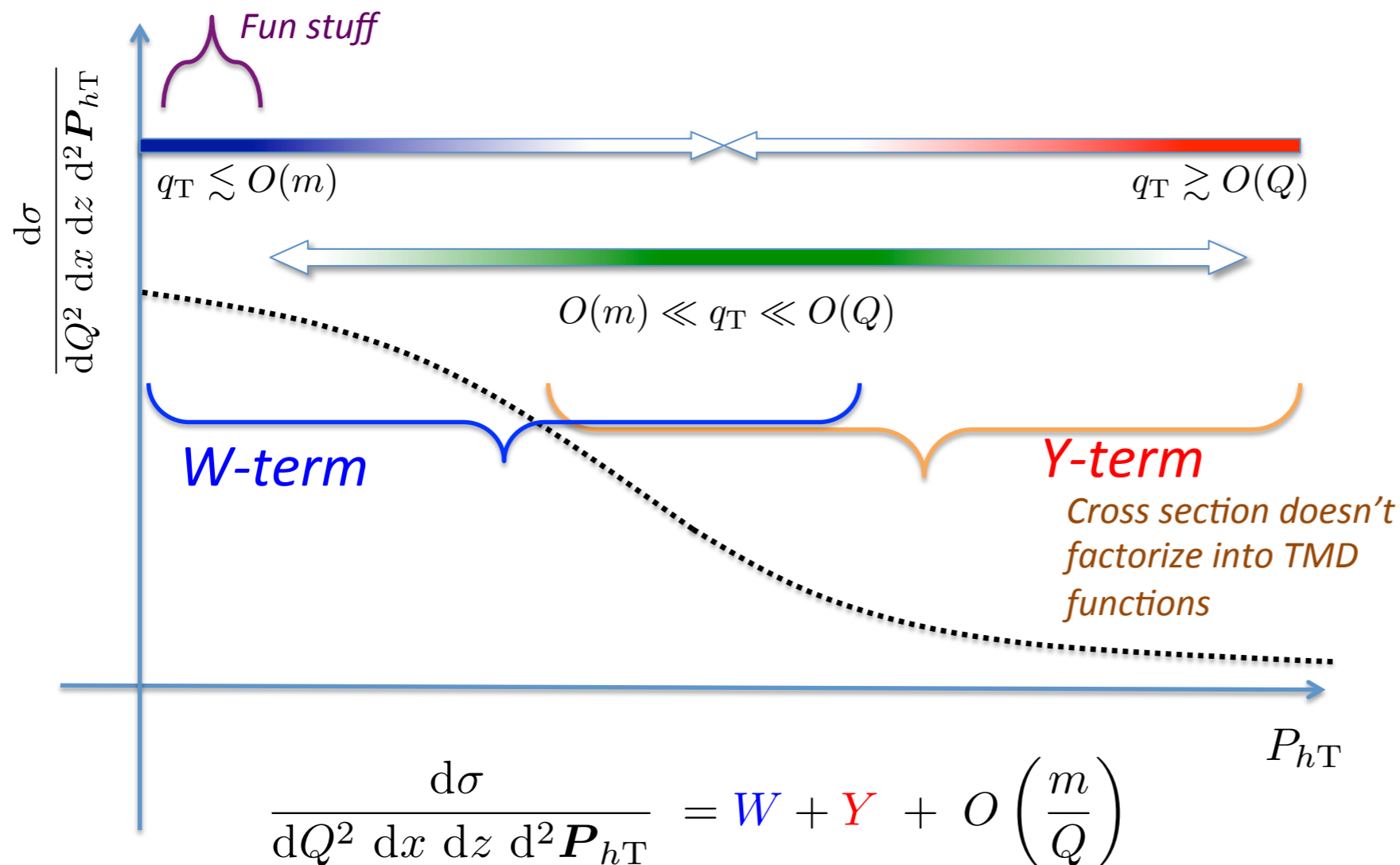
$$m \lesssim q_T \lesssim Q$$

Matching and $W + Y$ construction

- This was designed with the aim to have a formalism that is valid to leading power in m/Q uniformly in q_T , where m is a typical hadronic mass scale
- and where there is a broad intermediate range of transverse momentum characterized by $m \ll q_T \ll Q$
 - ♦ Collins Soper Serman NPB 1985
 - ♦ A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)

W + Y

From Ted Rogers

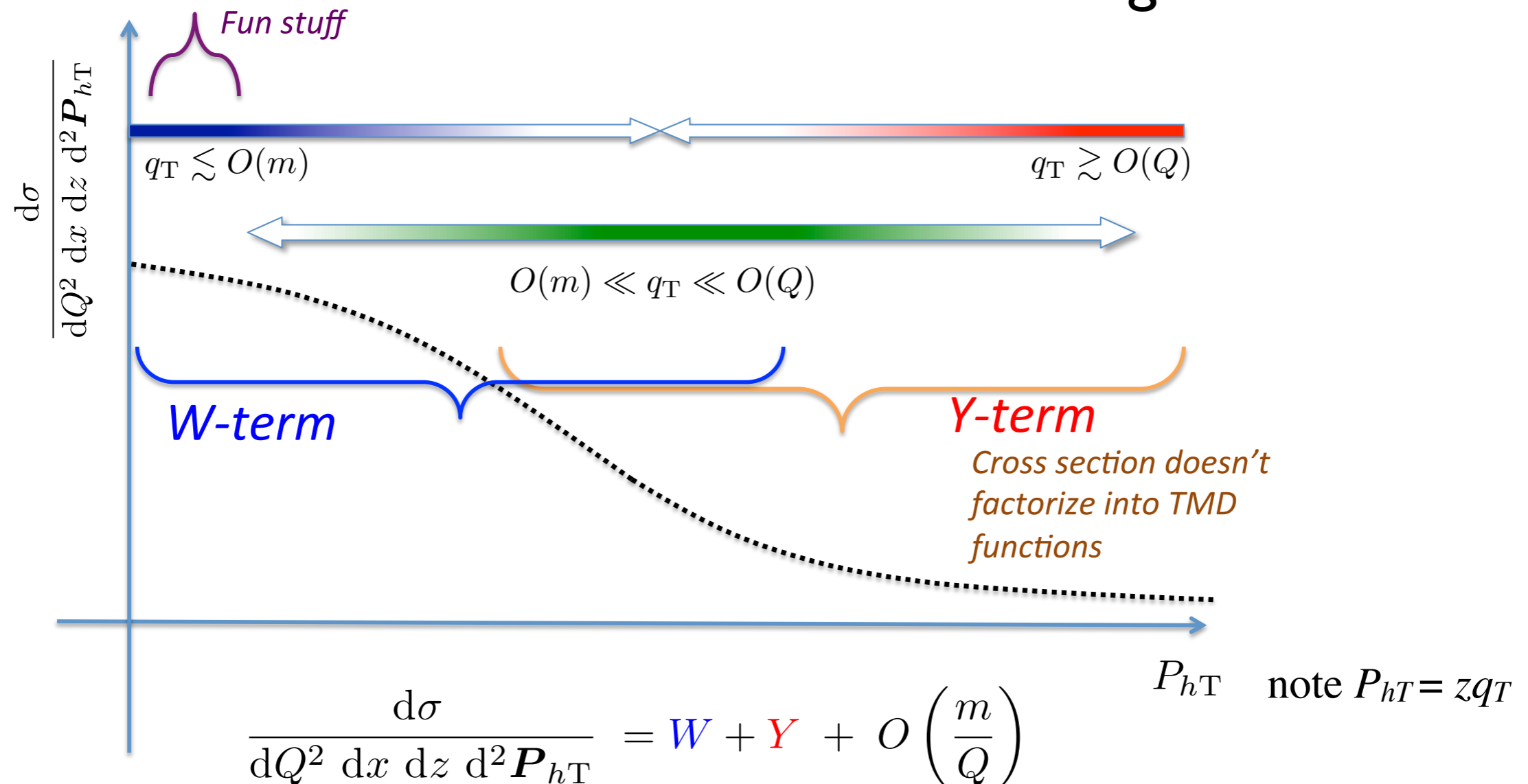


Matching and $W + Y$ construction

- However at lower phenomenologically interesting values of Q , neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

W + Y

From Ted Rogers



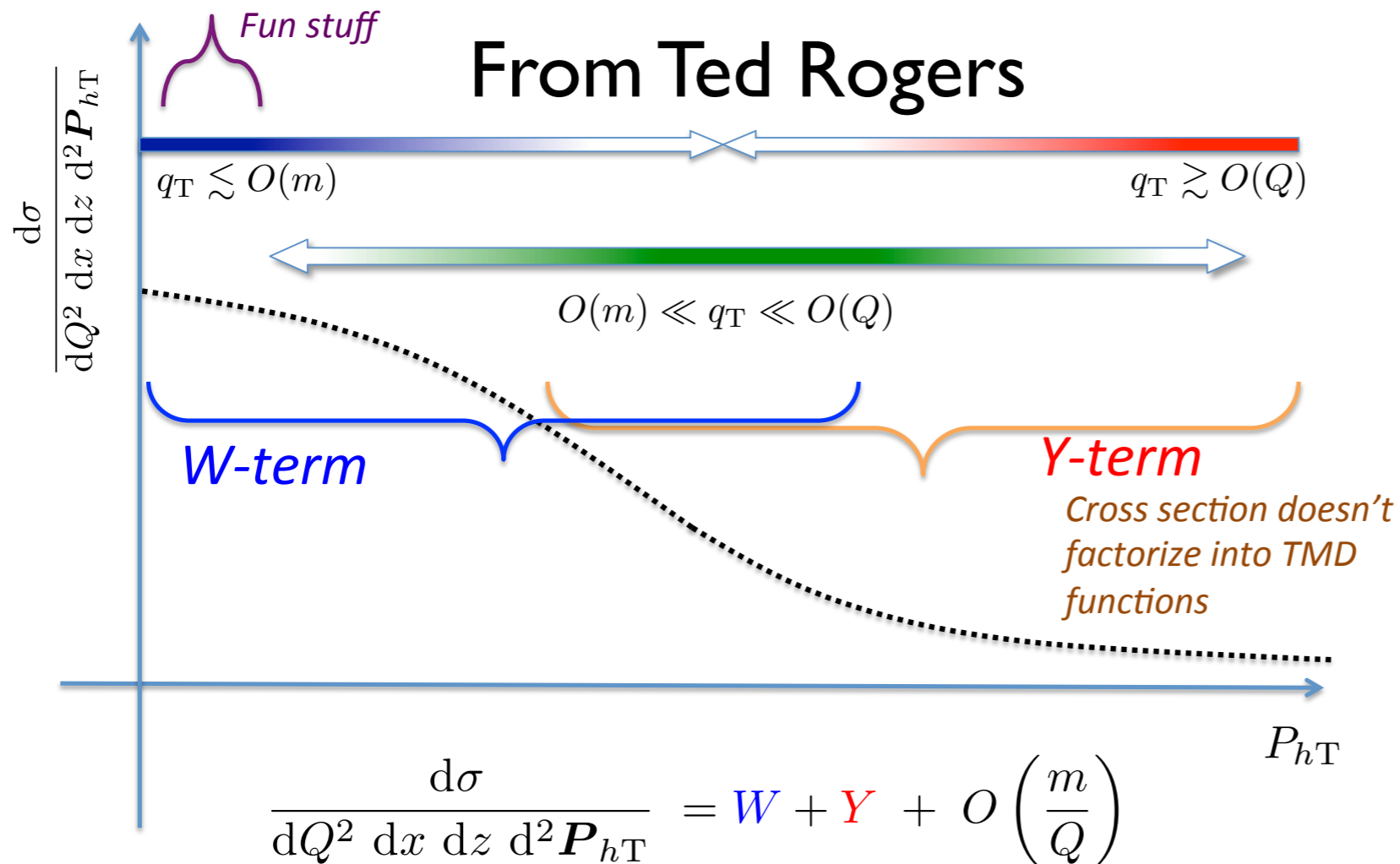
Matching and $W + Y$ construction

This impacts studies of non-perturbative nucleon structure @ COMPASS & JLAB

$$m \lesssim q_T \lesssim Q$$

W + Y

From Ted Rogers



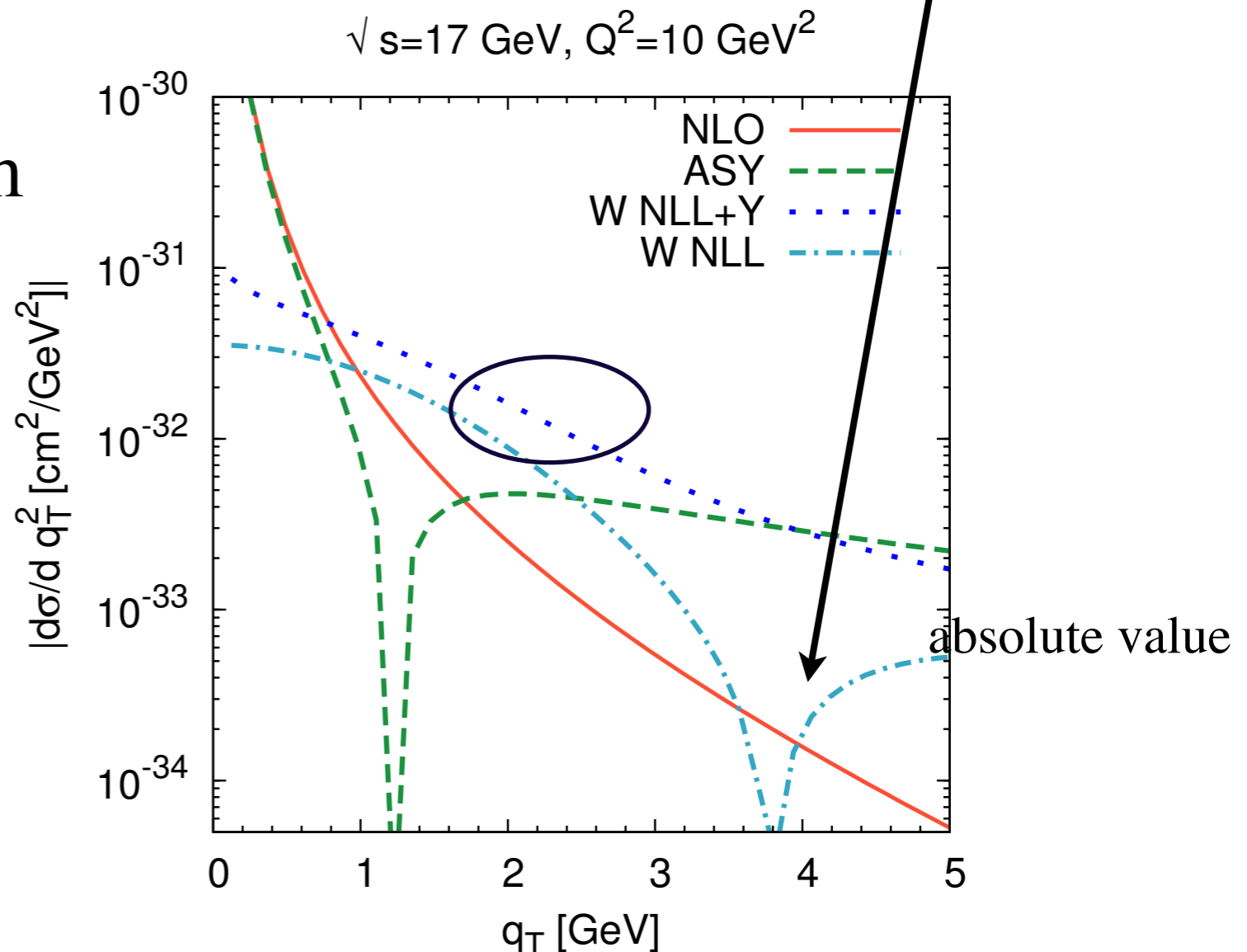
note $P_{hT} = zq_T$

Example

- When q_T is above some small fraction of Q , W deviates a lot from $\Gamma(q_T, Q)$
- Then it becomes negative and “asymptotes” to $\frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$
Nadowsly et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006),
- At large q_T $W+Y$ is then a difference of large terms and truncation errors can be augmented

Matching becomes a problem
COMPASS like energies

Boglione Prokudin et al. JHEP 2015



- To get a sense of source of truncation errors we further “unpack” $W+Y$ Construction





Review of Region Analysis “Approximators” W & Y Definitions

Original CSS definition of W is given by the instruction to carryout an approximation of the cs that is designed to be good in the region $q_T \ll Q$ up to powers of q_T/Q and m/Q

$$T_{TMD}\Gamma(q_T, Q) \approx \Gamma(q_T \ll Q, Q) + O\left(\frac{q_T}{Q}\right)^a \Gamma(q_T, Q) + O\left(\frac{m}{Q}\right)^{a'} \Gamma(q_T, Q)$$

$$W(q_T, Q) \equiv T_{TMD}\Gamma(q_T, Q)$$

Another approximator for the “region” of $q_T \sim Q$ defines FO up to powers of m/q_T

$$T_{coll}\Gamma(q_T, Q) \approx \Gamma(q_T \gtrsim Q, Q) + O\left(\frac{m}{q_T}\right)^b \Gamma(q_T, Q)$$

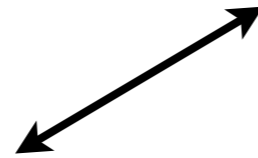
$$FO(q_T, Q) \equiv T_{coll}\Gamma(q_T, Q)$$



Region analysis $W + Y$ construction

Standard method to combine W & Y is to construct a sequence of nested subtractions. The smallest-size region is a neighborhood of $q_T = 0$, where T_{TMD} gives a very good approximation. So, one starts by adding and subtracting the T_{TMD} approximation.

$$\Gamma(q_T, Q) = T_{TMD}\Gamma(q_T, Q) + [\Gamma(q_T, Q) - T_{TMD}\Gamma(q_T, Q)]$$



- The error in the bracket is order $(q_T/Q)^a$ and is only unsuppressed at $q_T \gg m$
- One thus applies T_{coll} and used the fixed order (FO) perturbative expansion see CSS 1985 NPB and JCC Cambridge Press 2011 for details

Result is the combination

$$\Gamma(m \lesssim q_T \lesssim Q, Q) \approx T_{TMD}\Gamma(q_T, Q) + T_{coll} [\Gamma(q_T, Q) - T_{TMD}\Gamma(q_T, Q)] + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

$$\Gamma(m \lesssim q_T \lesssim Q, Q) \approx W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

$q_T/Q \ll 1$

$q_T \sim Q$ or $m/q_T \ll 1$

Definition of the Y term

$$Y(q_T, Q) \equiv T_{coll} \Gamma(q_T, Q) - T_{coll} T_{TMD} \Gamma(q_T, Q)$$

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

- It is the difference of the cross section calculated with collinear pdfs and ffs at fixed order FO and the asymptotic contribution of the cross section
- At small q_T the FO and ASY are dominated by the same diverging terms

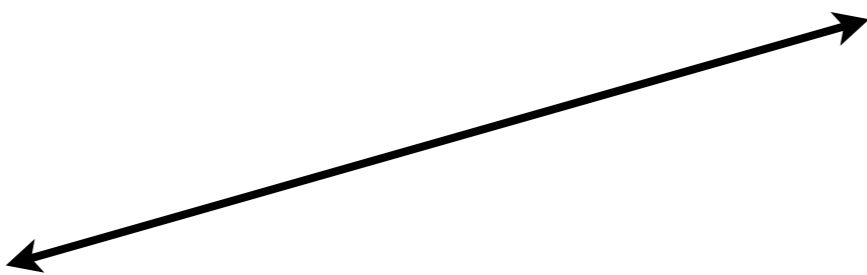
$$\frac{1}{q_T^2} \quad \text{and} \quad \frac{1}{q_T^2} \log \frac{Q^2}{q_T^2}$$

- thus its expected that the Y term is small or zero leaving

$$\Gamma(q_T \ll Q, Q) \approx W(q_T, Q)$$

The Asymptotic piece of the NLO cross section

$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$



$$\left(\frac{d\sigma_{BA}}{dx dz dQ^2 dq_T^2 d\phi} \right)_{\text{asym}} = \frac{\sigma_0 F_l}{S_{eA}} \frac{\alpha_s}{\pi} \frac{1}{2q_T^2} \frac{A_1(\psi, \phi)}{2\pi}$$

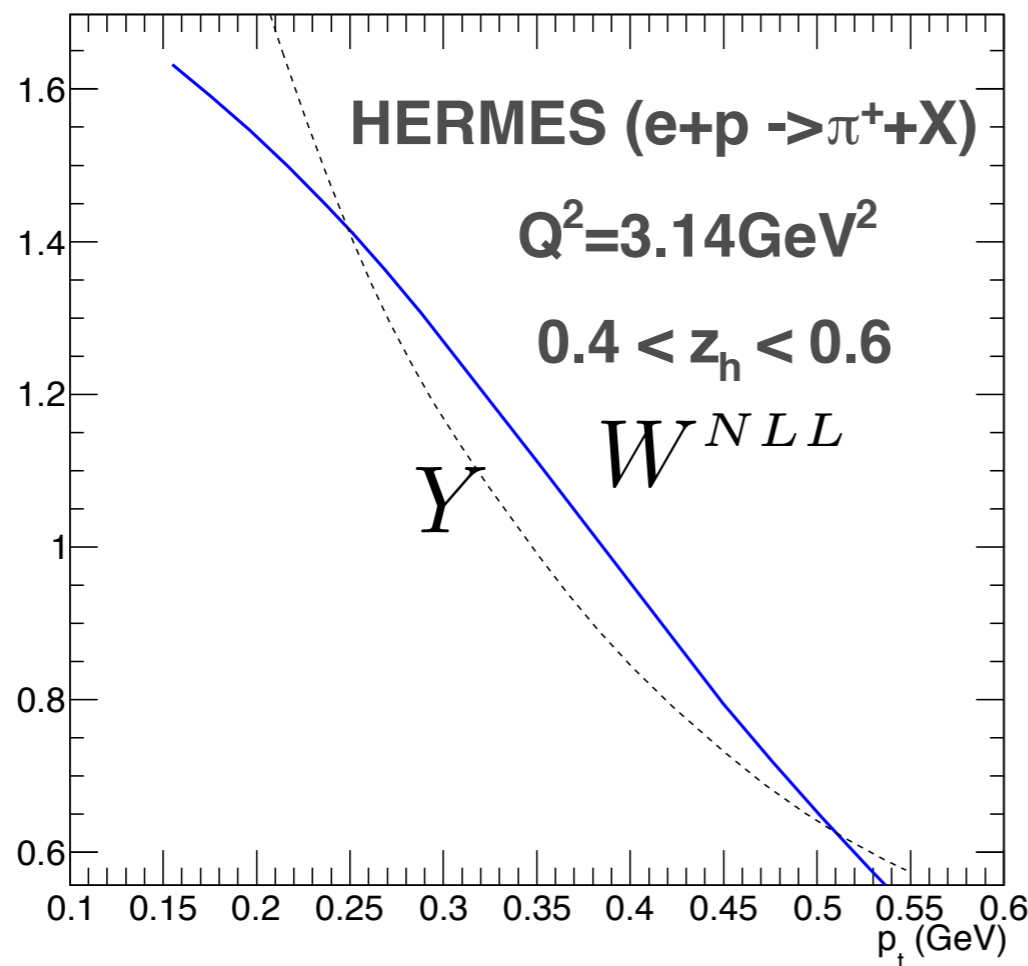
$$\times \sum_j e_j^2 \left[D_{B/j}(z, \mu) \{ (P_{qq} \otimes f_{j/A})(x, \mu) + (P_{qg} \otimes f_{g/A})(x, \mu) \} \right.$$

$$+ \{ (D_{B/j} \otimes P_{qq})(z, \mu) + (D_{B/g} \otimes P_{gq})(z, \mu) \} f_{j/A}(x, \mu)$$

$$\left. + 2D_{B/j}(z, \mu) f_{j/A}(x, \mu) \left\{ C_F \log \frac{Q^2}{q_T^2} - \frac{3}{2} C_F \right\} + \mathcal{O}\left(\frac{\alpha_s}{\pi}, q_T^2\right) \right].$$

- Nadowsky et al. PRD 1999, Y. Koike, J. Nagashima, and W. Vogelsang, Nucl. Phys. B744, 59 (2006),

- At small q_T the Y term is in principle suppressed: it is the difference of the FO perturbative calculation of the cross section and the asymptotic contribution of W for small q_T
- But again there can be a difference of of large terms and truncation errors are augmented: Here the Y term is larger than W ?!

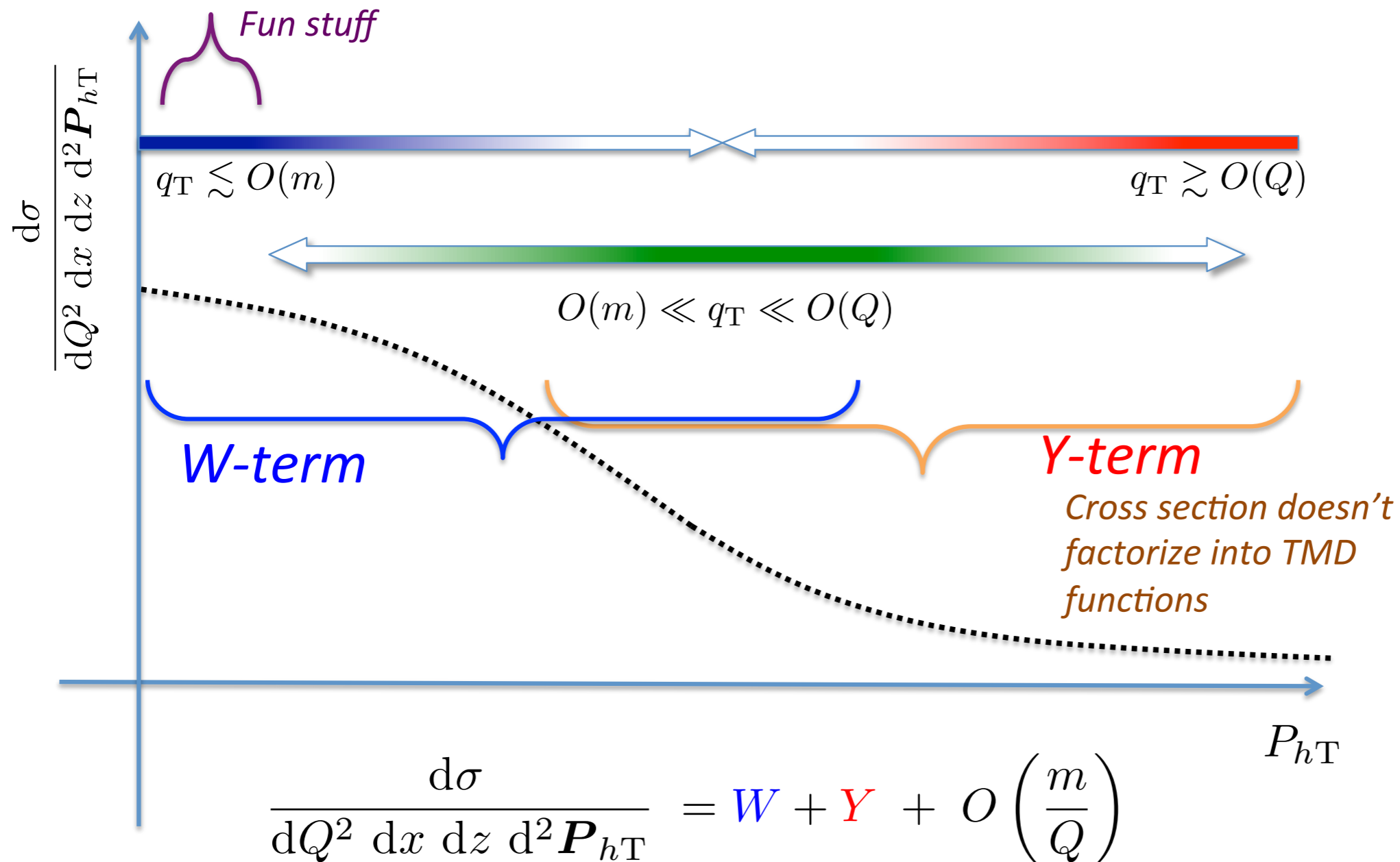


$$Y(q_T, Q) = FO(q_T, Q) - ASY(q_T, Q)$$

Region analysis $W + Y$ construction

- Thus the region *between* large and small q_T needs special treatment if errors are to be strictly power suppressed point-by-point in q_T .

W + Y



Extend formalism to

$$q_T \lesssim m \quad \text{and} \quad q_T \gtrsim Q$$

Extend formalism to

$$q_T \lesssim m$$

- For $q_T \lesssim m$ collinear factorization is not applicable for the differential cross section. But this region is actually where the W-term in has its highest validity. So one simply must ensure that the Y-term is sufficiently suppressed in Eq. (10) for $q_T \lesssim m$
- Modify Y to

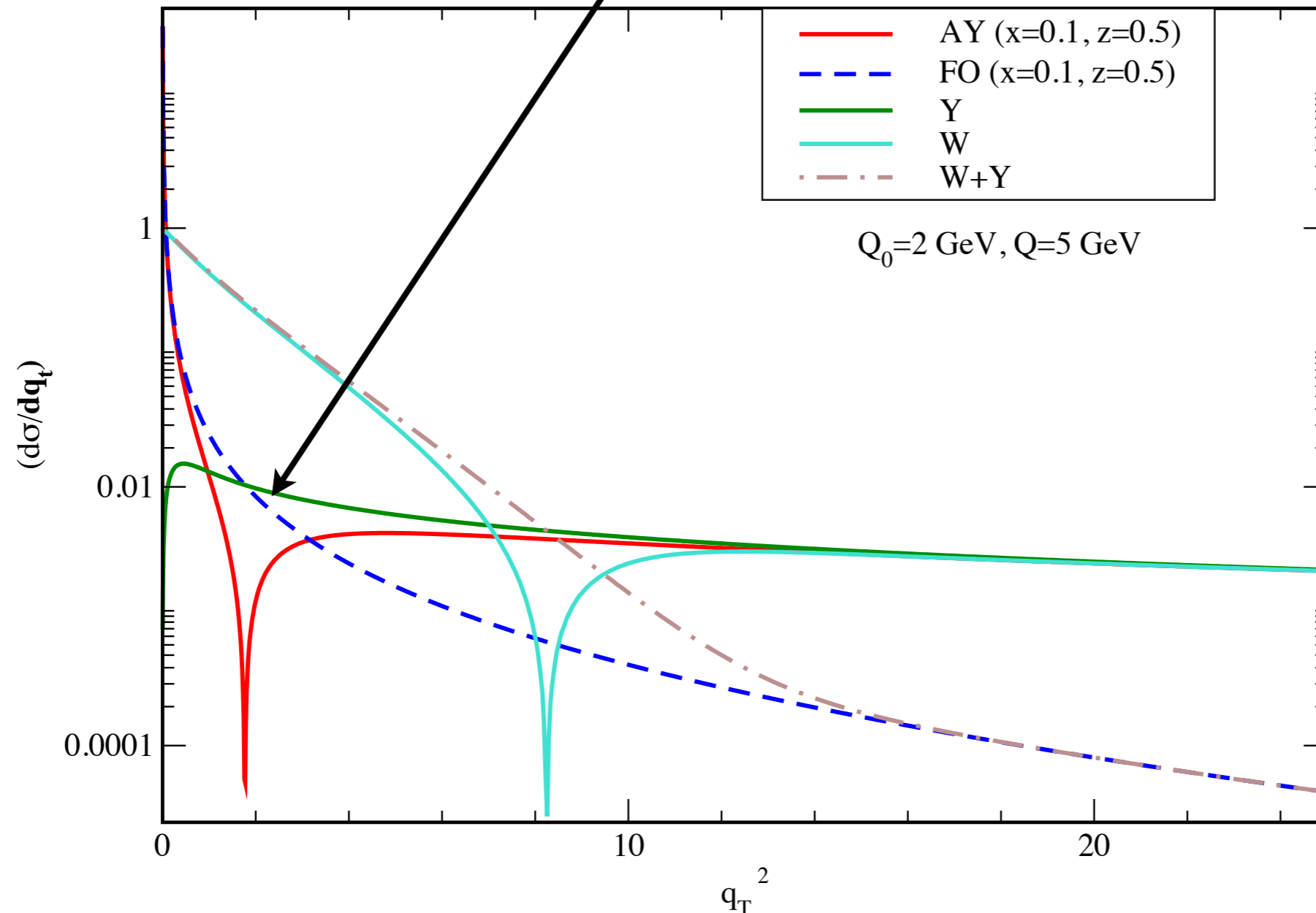
$$Y(q_T, Q) = \{FO(q_T, Q) - ASY(q_T, Q)\} X(q_T/\lambda)$$

with small q_T cutoff

$$X(q_T/\lambda) = 1 - \exp \{ -(q_T/\lambda)^{a_X} \}$$

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$\Gamma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$



Extend formalism to

$$q_T \gtrsim Q$$

Modification of the cross section leaves the standard treatment of TMD factorization only slightly modified.

In particular the op. definitions along with evolution properties are the same as in the usual formalism

We do this in two steps however now we need explicit expression for W from JCC formalism

see Collins Rogers PRD 2015

Summary of elements of TMD factorization

See talk of Zhongbo Kang

$$W(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}(b_T, Q)$$

- Factorization and TMD evolution in b_T space
- Solve the CSS & RG evolution Eqs for W term in SIDIS with “boundary condition” to freeze b_T above some b_{max} and with BCs

$$b_*(b_T) = \sqrt{\frac{b_T^2}{1 + b_T^2/b_{max}}}$$

$$\tilde{W}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}(b_*(b_T), Q) \tilde{W}_{NP}(b_T, Q; b_{max})$$

$$\tilde{W}_i^{OPE}(b_*(b_T), Q) = H_i(Q) \tilde{C}_{i/i'}^{pdf}(x_A/\hat{x}, b_* b_*) \otimes \tilde{f}_{i'/A}(\hat{x}, \mu_{b_*}) \tilde{C}_{j'/i}^{ff}(z_B/\hat{z}, b_*) \otimes \tilde{d}_{B/i'}(\hat{z}, \mu_b) e^{-S^{pert}(b_*, Q)}$$

Collinear pdfs

$$\tilde{W}_{NP}(b_T, Q; b_{max}) = e^{-S_{NP}(b_T, Q; b_{max})}$$

$$S_{NP}(b_T, Q; b_{max}) = g_A(x_A, b_T; b_{max}) + g_B(z_B, b_T; b_{max}) - 2g_K(b_T; b_{max}) \ln \left(\frac{Q}{Q_0} \right)$$

Fourier Transforms of TMDs and universal soft function g_k

Two modifications

a) Introduce small b-cuttoff

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2 / (C_5 Q)} \implies b_c(0) \sim 1/Q$$

b) Introduce large q_T -cuttoff so that W_{New} vanishes at large q_T

$$\Xi\left(\frac{q_T}{Q}, \eta\right) = \exp\left[-\left(\frac{q_T}{\eta Q}\right)^{a_\Xi}\right]$$

$$\tilde{W}_{\text{New}}(q_T, Q; \eta, C_5) = \Xi\left(\frac{q_T}{Q}, \eta\right) \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}(b_*(b_c(b_T)), Q) \tilde{W}_{NP}(b_c(b_T), Q; b_{\text{max}})$$

$$b_*(b_c(b_T)) \longrightarrow \begin{cases} b_{\text{min}} & b_T \ll b_{\text{min}} \\ b_T & b_{\text{min}} \ll b_T \ll b_{\text{max}} \\ b_{\text{max}} & b_T \gg b_{\text{max}} \end{cases}$$

i) Semi-inclusive to Collinear
integrate over q_T

- Parton Model W-term

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2 k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$
$$\int d^2 q_T W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

- Standard CSS W-term

$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$
$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

Phys.Rev. D 94 (2016) J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

See appendix for details **Phys.Rev. D 94 (2016)**

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$$W_{CSS}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{CSS}(b_T, Q)$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = \int \delta^2(b_T) b_T \times \text{logarithmic corrections}$$

$$\int d^2 q_T W_{CSS}(q_T, Q) = 0 \quad !$$

For details Phys.Rev. D 94 (2016)

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$$W_{New}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T \cdot b_T} \tilde{W}_{New}(b_T, Q)$$

$$\int d^2 q_T W_{New}(q_T, Q) = \tilde{W}(b_{c \min}, Q) \quad \text{has a normal collinear factorization in terms of collinear pdfs}$$

$$\int d^2 q_T W_{New}(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) + O(\alpha_s(Q))$$

Has implications for modeling TMD and fitting

Large q_T -cutoff so on W_{New}
vanishes at large q_T

b) Introduce large q_T -cutoff so that
 W_{New} vanishes at large q_T

$$\Xi \left(\frac{q_T}{Q}, \eta \right) = \exp \left[- \left(\frac{q_T}{\eta Q} \right)^{a_\Xi} \right]$$

$$\tilde{W}_{\text{New}}(q_T, Q; \eta, C_5) = \Xi \left(\frac{q_T}{Q}, \eta \right) \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE} (b_*(b_c(b_T)), Q) \tilde{W}_{NP}(b_c(b_T), Q; b_{\text{max}})$$

$$b_*(b_c(b_T)) \longrightarrow \begin{cases} b_{\text{min}} & b_T \ll b_{\text{min}} \\ b_T & b_{\text{min}} \ll b_T \ll b_{\text{max}} \\ b_{\text{max}} & b_T \gg b_{\text{max}} . \end{cases}$$

Now Y term is further modified

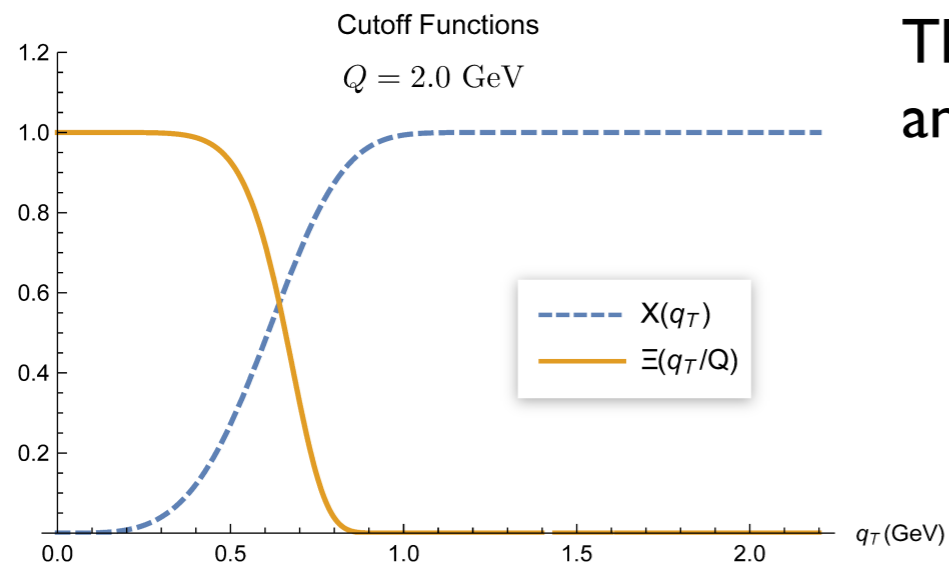
$$\begin{aligned} Y_{New}(q_T, Q) &= [T_{coll}\Gamma(q_T, Q) - T_{coll}T_{TMD}^{New}\Gamma(q_T, Q)] X(q_T/\lambda) \\ &= [FO(q_T, Q) - ASY_{New}(q_T, Q)] X(q_T/\lambda) \end{aligned}$$

Putting all together

$$\Gamma(q_T, Q) \approx T_{TMD}^{New} \Gamma(q_T, Q) + T_{coll} [\Gamma(q_T, Q) - T_{TMD}^{New} \Gamma(q_T, Q)] + \mathcal{O}\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

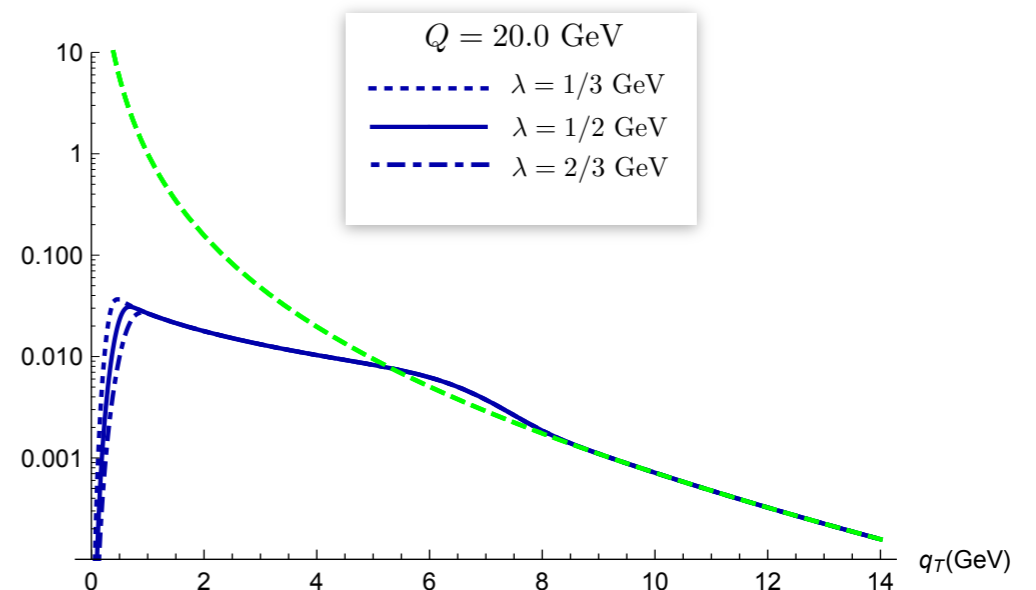
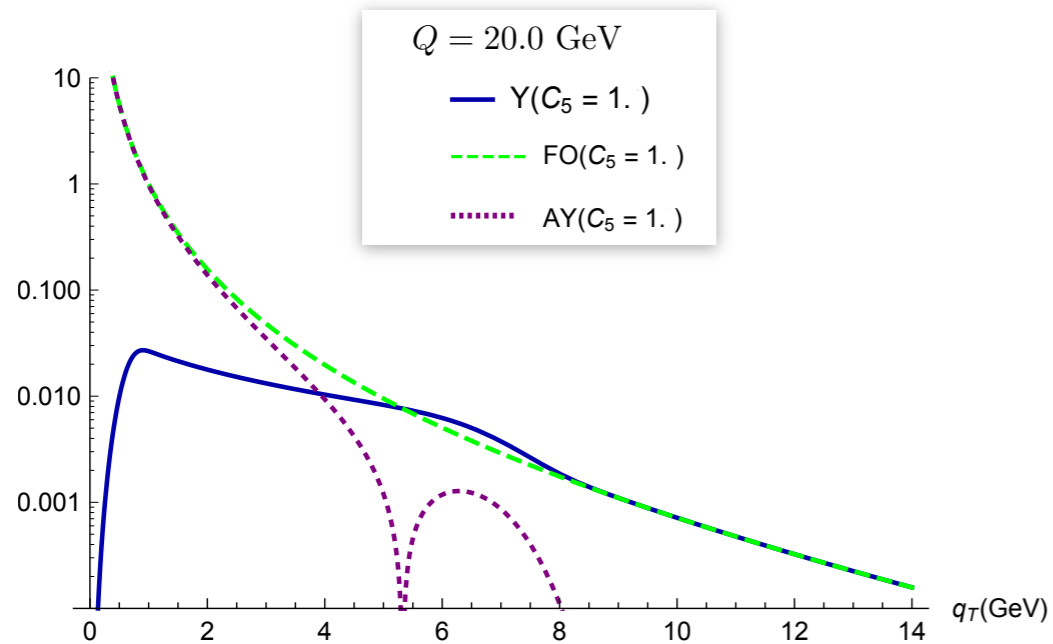
$$\Gamma(q_T, Q) \approx W_{New}(q_T, Q) + Y_{New}(q_T, Q) + \mathcal{O}\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

Putting all together



The cutoff functions in for low q_T/λ (blue dashed line) and large q_T/Q (brown solid line) for $Q = 20.0 \text{ GeV}$

Y-term



Comments

- ◆ The standard $W + Y$ prescription was arranged to apply also for intermediate q_T ; in particular it keeps full accuracy when $m \ll q_T \ll Q$, a situation in which both pure TMD and pure collinear factorization have degraded accuracy
- ◆ It also did not specifically address the issue of matching to collinear factorization for the cross section integrated over q_T .
- ◆ With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of $1/Q$.
- ◆ Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used.
- ◆ This work has dealt only with unpolarized cross sections.
- ◆ We are studying the analogous topic applied to polarized phenomena.
- ◆ This is central to the EIC and studying the 3-D momentum and spatial structure of the nucleon and further exploring the connection between TMD and collinear factorization

EXTRA Slides

Expression for $W(b_c, Q)$

$$\begin{aligned}
 \tilde{W}(b_c(b_T), Q) = & H(\mu_Q, Q) \sum_{j' i'} \int_{x_A}^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{j/j'}^{\text{pdf}}(x_A/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) f_{j'/A}(\hat{x}; \bar{\mu}) \times \\
 & \times \int_{z_B}^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}_{i'/j}^{\text{ff}}(z_B/\hat{z}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) d_{B/i'}(\hat{z}; \bar{\mu}) \times \\
 & \times \exp \left\{ \ln \frac{Q^2}{\bar{\mu}^2} \tilde{K}(b_*(b_c(b_T)); \bar{\mu}) + \int_{\bar{\mu}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\} \\
 & \times \exp \left\{ -g_A(x_A, b_c(b_T); b_{\text{max}}) - g_B(z_B, b_c(b_T); b_{\text{max}}) - 2g_K(b_c(b_T); b_{\text{max}}) \ln \left(\frac{Q}{Q_0} \right) \right\}
 \end{aligned}$$

$$b_*(b_c(b_T)) \longrightarrow \begin{cases} b_{\text{min}} & b_T \ll b_{\text{min}} \\ b_T & b_{\text{min}} \ll b_T \ll b_{\text{max}} \\ b_{\text{max}} & b_T \gg b_{\text{max}} . \end{cases}$$

Region analysis $W + Y$ construction

- If $q_T \ll m$ and $q_T \ll Q$ were the only regions of interest, then the TTMD and Tcoll approximators would be sufficient. One could simply calculate using fixed order collinear factorization for the large q_T -dependence and TMD factorization for small q_T -dependence.

$$W(q_T, Q) \equiv T_{TMD} \Gamma(q_T, Q)$$

or ...

$$FO(q_T, Q) \equiv T_{coll} \Gamma(q_T, Q)$$

- A reasonable description of the full transverse momentum dependence would be obtained by simply interpolating between the two descriptions

Region analysis $W + Y$ construction

$$\Gamma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c \Gamma(q_T, Q)$$

- The error estimates in Eq. are not applicable outside this range i.e., they must not be applied when $q_T \gg Q$ or $q_T \ll m$
- This is because W & Y were extracted from the leading power of expansions in relatively small kinematic variables q_T/Q and m/q_T to give
- However for $m < q_T < O(Q)$, the cross section given by $W + Y$ should appropriately match FO collinear perturbation theory calculations for large transverse momentum.